11th Thailand Mathematical Olympiad
Khon Kaen University, Khon Kaen
13 May 2014

Day 1
Time: 4.5 hours

1. Let $\triangle ABC$ be an isosceles triangle with $\angle BAC = 100°$. Let D, E be points on ray $\overrightarrow{AB}$ so that $BC = AD = BE$. Show that $BC \cdot DE = BD \cdot CE$.

2. Determine all functions $f : \mathbb{R} \to \mathbb{R}$ satisfying

$$f(xy - 1) + f(x)f(y) = 2xy - 1$$

   for all real numbers x, y.

3. Let M and N be positive integers. Pisut walks from point $(0, N)$ to point $(M, 0)$ in steps so that

   - each step has unit length and is parallel to either the horizontal or the vertical axis, and
   - each point (x, y) on the path has nonnegative coordinates, i.e. $x, y \geqslant 0$.

   During each step, Pisut measures his distance from the axis parallel to the direction of his step; if after the step he ends up closer from the origin (compared to before the step) he records the distance as a positive number, else he records it as a negative number.

   Prove that, after Pisut completes his walk, the sum of the signed distances Pisut measured is zero.

4. Find all polynomials $P(x)$ with integer coefficients such that $P(n) \mid 2557^n + 213 \times 2014$ for all positive integers n.

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Day 2
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5. Determine the maximal value of k such that the inequality

$$\left(k+\frac{a}{b}\right)\left(k+\frac{b}{c}\right)\left(k+\frac{c}{a}\right) \leqslant \left(\frac{a}{b}+\frac{b}{c}+\frac{c}{a}\right)\left(\frac{b}{a}+\frac{c}{b}+\frac{a}{c}\right)$$

holds for all positive reals a, b, c.

6. Find all primes p such that $2p^2 - 3p - 1$ is a positive perfect cube.

7. Let $\square ABCD$ be a convex quadrilateral with shortest side AB and longest side CD, and suppose that $AB < CD$. Show that there is a point $E \neq C, D$ on segment CD with the following property:

   For all points $P \neq E$ on side CD, if we define O_1 and O_2 to be the circumcenters of $\triangle APD$ and $\triangle BPE$ respectively, then the length of O_1O_2 does not depend on P.

8. Let n be a positive integers. A collection of cards, each numbered with a positive integer, is created so that

   - the number on each card is of the form $m!$ for some positive integer m, and
   - for all positive integers $t \leqslant n!$, it is possible to choose some cards from the collection so that the sum of numbers on the chosen cards is exactly t.

   What is the minimum possible number of cards in the collection?