Thailand October Camp 2019
IPST, Bangkok
21 October 2019

TSTST 1 Day 1
Time: 4.5 hours

1. Let ABC be a triangle. Circle Γ passes through point A, meets segments AB and AC again at D and E respectively, and intersects segment BC at F and G such that F lies between B and G. The tangent to circle (BDF) at F and the tangent to circle (CEG) at G meet at T. Suppose that points A and T are distinct. Prove that line AT is parallel to BC.

2. Let x, y, z be positive real numbers such that $x^2 + y^2 + z^2 = 3$. Prove that

$$\frac{x+1}{z+x+1} + \frac{y+1}{x+y+1} + \frac{z+1}{y+z+1} \geqslant \frac{\left(xy+yz+zx+\sqrt{xyz}\right)^2}{(x+y)(y+z)(z+x)}.$$

3. Find all pairs of positive integers (m, n) satisfying the equation

$$m! + n! = m^n + 1.$$

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TSTST 1 Day 2
Time: 4.5 hours

4. A 1×2019 board is filled with numbers $1, 2, \ldots, 2019$ in an increasing order. In each step, three consecutive tiles are selected, then one of the following operations is performed:

   (i) the number in the middle is increased by 2 and its neighbors are decreased by 1, or

   (ii) the number in the middle is decreased by 2 and its neighbors are increased by 1.

   After several such operations, the board again contains all the numbers $1, 2, \ldots, 2019$. Prove that each number is in its original position.

5. Let $\{a_n\}$ be a sequence of positive integers such that $a_{n+1} = a_n^2 + 1$ for all $n \geqslant 1$. Prove that there is no positive integer N such that

$$\prod_{k=1}^{n}(a_k^2 + a_k + 1)$$

   is a perfect square.

6. Prove that the unit square can be tiled with rectangles (not necessarily of the same size) similar to a rectangle of size $1 \times (3 + \sqrt[3]{3})$.

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TSTST 2 Day 1
Time: 4.5 hours

1. Find all functions $f : \mathbb{R} \to \mathbb{R}$ satisfying

$$f\left(\max\{x, y\} + \min\{f(x), f(y)\}\right) = x + y$$

for all $x, y \in \mathbb{R}$.

2. For any positive integer $m \geqslant 2$, let $p(m)$ be the smallest prime dividing m and $P(m)$ be the largest prime dividing m. Let C be a positive integer. Define sequences $\{a_n\}$ and $\{b_n\}$ by $a_0 = b_0 = C$ and, for each positive integer k such that $a_{k-1} \geqslant 2$,

$$a_k = a_{k-1} - \frac{a_{k-1}}{p(a_{k-1})};$$

and, for each positive integer k such that $b_{k-1} \geqslant 2$,

$$b_k = b_{k-1} - \frac{b_{k-1}}{P(b_{k-1})}.$$

It is easy to see that both $\{a_n\}$ and $\{b_n\}$ are finite sequences which terminate when they reach the number 1.

Prove that the numbers of terms in the two sequences are always equal.

3. Let ABC be an acute triangle and Γ be its circumcircle. Line ℓ is tangent to Γ at A and let D and E be distinct points on ℓ such that $AD = AE$. Suppose that B and D lie on the same side of line AC. The circumcircle Ω_1 of $\triangle ABD$ meets AC again at F. The circumcircle Ω_2 of $\triangle ACE$ meets AB again at G. The common chord of Ω_1 and Ω_2 meets Γ again at H. Let K be the reflection of H across line BC and let L be the intersection of BF and CG. Prove that A, K and L are collinear.

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4 November 2019

TSTST 2 Day 2
Time: 4.5 hours

4. Does there exist a set S of positive integers satisfying the following conditions?
   (i) S contains 2020 distinct elements;
   (ii) the number of distinct primes in the set $\{\gcd(a,b) : a,b \in S, a \neq b\}$ is exactly 2019; and
   (iii) for any subset A of S containing at least two elements, $\displaystyle\sum_{a,b\in A;\ a<b} ab$ is not a prime power.

5. Let P be an interior point of a circle Γ centered at O where $P \neq O$. Let A and B be distinct points on Γ. Lines AP and BP meet Γ again at C and D, respectively. Let S be any interior point on line segment PC. The circumcircle of $\triangle ABS$ intersects line segment PD at T. The line through S perpendicular to AC intersects Γ at U and V. The line through T perpendicular to BD intersects Γ at X and Y. Let M and N be the midpoints of UV and XY, respectively. Let AM and BN meet at Q. Suppose that AB is not parallel to CD. Show that P, Q, and O are collinear if and only if S is the midpoint of PC.

6. A nonempty set S is called *Bally* if for every $m \in S$, there are fewer than $\frac{1}{2}m$ elements of S which are less than m. Determine the number of Bally subsets of $\{1, 2, \ldots, 2020\}$.