Thailand October Camp 2018
IPST, Bangkok
19 October 2018

Day 1
Time: 4.5 hours

1. Let 2561 given points on a circle be colored either red or green. In each step, all points are recolored simultaneously in the following way: if both direct neighbors of a point P have the same color as P, then the color of P remains unchanged, otherwise P obtains the other color. Starting with the initial coloring F_1, we obtain the colorings $F_2, F_3, \ldots$ after several recoloring steps. Determine the smallest number n such that, for any initial coloring F_1, we must have $F_n = F_{n+2}$.

2. Let Ω be the inscribed circle of a triangle $\triangle ABC$. Let D, E and F be the tangency points of Ω and the sides BC, CA and AB, respectively, and let AD, BE and CF intersect Ω at K, L and M, respectively, such that D, E, F, K, L and M are all distinct. The tangent line of Ω at K intersects EF at X, the tangent line of Ω at L intersects DE at Y, and the tangent line of Ω at M intersects DF at Z. Prove that X,Y and Z are collinear.

   Hint as given on the test: notice the asymmetry in defining the points X, Y and Z.

3. Find all functions $f : \mathbb{Z} \to \mathbb{Z}$ satisfying

   (i) $f\big(f(m) + n\big) + 2m = f(n) + f(3m)$ for every $m, n \in \mathbb{Z}$,

   (ii) there exists a $d \in \mathbb{Z}$ such that $f(d) - f(0) = 2$, and

   (iii) $f(1) - f(0)$ is even.

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Day 2
Time: 4.5 hours

1. Find all primes p such that $(p-3)^p + p^2$ is a perfect square.

2. Let $a, b, c \in (0, \frac{4}{3})$ and $a+b+c=3$. Prove that

$$\frac{4abc}{(a+b)(a+c)} + \frac{(a+b)^2+(a+c)^2}{(a+b)+(a+c)} \leqslant \sum_{\text{cyc}} \frac{1}{a^2(3b+3c-5)}.$$

3. Let ABC be an acute triangle with AX, BY and CZ as its altitudes.

   - Line ℓ_A, which is parallel to YZ, intersects CA at A_1 between C and A, and intersects AB at A_2 between A and B.
   - Line ℓ_B, which is parallel to ZX, intersects AB at B_1 between A and B, and intersects BC at B_2 between B and C.
   - Line ℓ_C, which is parallel to XY, intersects BC at C_1 between B and C, and intersects CA at C_2 between C and A.

   Suppose that the perimeters of the triangles $\triangle AA_1A_2, \triangle BB_1B_2$ and $\triangle CC_1C_2$ are equal to $CA+AB$,$AB+BC$ and $BC+CA$, respectively. Prove that ℓ_A, ℓ_B and ℓ_C are concurrent.

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Day 3
Time: 4.5 hours

1. Let $\{x_i\}_{i=1}^{\infty}$ and $\{y_i\}_{i=1}^{\infty}$ be sequences of real numbers such that $x_1 = y_1 = \sqrt{3}$,

$$x_{n+1} = x_n + \sqrt{1 + x_n^2} \quad \text{and} \quad y_{n+1} = \frac{y_n}{1 + \sqrt{1 + y_n^2}}$$

for all $n \geqslant 1$. Prove that $2 < x_n y_n < 3$ for all $n > 1$.

2. Find all nonnegative integers x, y, z satisfying the equation

$$2^x + 31^y = z^2.$$

3. Let $n \geqslant 2$ be an integer. Determine the number of terms in the polynomial

$$\prod_{1 \leqslant i < j \leqslant n} (x_i + x_j)$$

whose coefficients are odd integers.