Thailand October Camp 2017
IPST, Bangkok
20 October 2017

Day 1 Morning
Time: 3 hours

Each problem is worth 5 points.

1. Prove that any rational $r \in (0,1)$ can be written uniquely in the form
$$r = \frac{a_1}{1!} + \frac{a_2}{2!} + \frac{a_3}{3!} + \cdots + \frac{a_k}{k!}$$
where a_i's are nonnegative integers with $a_i \leqslant i-1$ for all i.

2. 9 horizontal and 9 vertical lines are drawn through a square. Prove that it is possible to select 20 rectangles so that the sides of each rectangle is a segment of one of the given lines (including the sides of the square), and for any two of the 20 rectangles, it is possible to cover one of them with the other (rotations are allowed).

3. Circles O_1, O_2 intersects at A, B. The circumcircle of O_1BO_2 intersects O_1, O_2 and line AB at R, S, T respectively. Prove that $TR = TS$

4. *(IMO 1980/2)* Define the numbers $a_0, a_1, \ldots, a_n$ in the following way:
$$a_0 = \frac{1}{2}, \quad a_{k+1} = a_k + \frac{a_k^2}{n} \quad (n > 1, k = 0, 1, \ldots, n-1).$$
Prove that $1 - \frac{1}{n} < a_n < 1$.

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Day 1 Afternoon
Time: 3 hours

Each problem is worth 5 points.

5. Find all triples of real numbers (a, b, c) satisfying

$$a + b + c = 14, \quad a^2 + b^2 + c^2 = 84, \quad a^3 + b^3 + c^3 = 584.$$

6. In $\triangle ABC$, $AB < AC$ and $\angle BAC = 90^\circ$. The perpendicular bisector of BC cut AC at K, and the perpendicular bisector of BK cut AB at L. If CL bisects $\angle ACB$, what are the possible values of $\angle ABC$?

7. Evaluate $\displaystyle\sum_{n=2017}^{2030} \sum_{k=1}^{n} \left\{ \frac{\binom{n}{k}}{2017} \right\}$.

8. There are n vertices and $m > n$ edges in a graph. Each edge is colored either red or blue. In each year, we are allowed to choose a vertex and flip the color of all edges incident to it. Prove that there is a way to color the edges (initially) so that they will never all have the same color.

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Day 2
Time: 4.5 hours

Each problem is worth 7 points.

1. Find all polynomials $P(x)$ with real coefficients satisfying: $P(2017) = 2016$ and
$$(P(x)+1)^2 = P(x^2+1).$$

2. There are three sticks, each of which has an integer length which is at least n; the sum of their lengths is $n(n+1)/2$.

   Prove that it is possible to break the sticks (possibly several times) so that the resulting sticks have length $1, 2, \ldots, n$.

   Note: a stick of length $a+b$ can be broken into sticks of lengths a and b.

3. Let BC be a chord not passing through the center of a circle ω. Point A varies on the major arc BC. Let E and F be the projection of B onto AC, and of C onto AB respectively. The tangents to the circumcircle of $\triangle AEF$ at E, F intersect at P.

   (a) Prove that P is independent of the choice of A.

   (b) Let H be the orthocenter of $\triangle ABC$, and let T be the intersection of EF and BC. Prove that $TH \perp AP$.

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Day 3
Time: 4.5 hours

Each problem is worth 7 points.

1. Let P be a given quadratic polynomial. Find all functions $f : \mathbb{R} \to \mathbb{R}$ such that

$$f(x + y) = f(x) + f(y) \text{ and } f(P(x)) = f(x) \text{ for all } x, y \in \mathbb{R}.$$

2. In triangle $\triangle ABC$, $\angle BAC = 135^\circ$. M is the midpoint of BC, and $N \neq M$ is on BC such that $AN = AM$. The line AM meets the circumcircle of $\triangle ABC$ at D. Point E is chosen on segment AN such that $AE = MD$. Show that $ME = BC$.

3. Find all pairs of integers $m, n \geqslant 2$ such that

$$n \mid 1 + m^{3^n} + m^{2\cdot 3^n}.$$