Thailand October Camp 2016
IPST, Bangkok
18 October 2016

Day 1 Morning
Time: 3 hours

1. In $\triangle ABC$, D, E, F are the midpoints of AB, BC, CA respectively. Denote by O_A, O_B, O_C the incenters of $\triangle ADF, \triangle BED, \triangle CFE$ respectively. Prove that O_AE, O_BF, O_CD are concurrent.

2. Let $m, n \in \mathbb{N}$ satisfy
$$\phi(5^m - 1) = 5^n - 1.$$
Prove that $\gcd(m, n) \neq 1$.

3. In $\triangle ABC$ with $AB > AC$, the tangent to the circumcircle at A intersects line BC at P. Let Q be the point on AB such that $AQ = AC$, and A lies between B and Q. Let R be the point on ray AP such that $AR = CP$. Let X, Y be the midpoints of AP, CQ respectively. Prove that $CR = 2XY$.

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Day 1 Afternoon
Time: 3 hours

4. Suppose that m, n, k are positive integers satisfying

$$3mk = (m+3)^n + 1.$$

Prove that k is odd.

5. Let ω_1, ω_2 be two circles with different radii, and let H be the exsimilicenter of the two circles. A point X outside both circles is given. The tangents from X to ω_1 touch ω_1 at P, Q, and the tangents from X to ω_2 touch ω_2 at R, S. If PR passes through H and is not a common tangent line of ω_1, ω_2, prove that QS also passes through H.

6. A and B plays a game, with A choosing a positive integer $n \in \{1, 2, \dots, 1001\} = S$. B must guess the value of n by choosing several subsets of S, then A will tell B how many subsets n is in. B will do this three times selecting k_1, k_2 then k_3 subsets of S each.

   What is the least value of $k_1 + k_2 + k_3$ such that B has a strategy to correctly guess the value of n no matter what A chooses?

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Day 2
Time: 4.5 hours

1. *Submissions for each subproblem (1.1-1.3) must be written within a single page.*

   1.1 Let $f(A)$ denote the difference between the maximum value and the minimum value of a set A. Find the sum of $f(A)$ as A ranges over the subsets of $\{1, 2, ..., n\}$

   1.2 All cells of an 8×8 board are initially white. A move consists of flipping the color (white to black or vice versa) of cells in a 1×3 or 3×1 rectangle. Determine whether there is a finite sequence of moves resulting in the state where all 64 cells are black.

   1.3 Prove that for all positive integers m, there exists a positive integer n such that the set $\{n, n+1, n+2, \ldots, 3n\}$ contains exactly m perfect squares.

2. Let f, g be bijections on $\{1, 2, 3, ..., 2016\}$. Determine the value of

$$\sum_{i=1}^{2016} \sum_{j=1}^{2016} [f(i) - g(j)]^{2559}.$$

3. Let f be a function on a set X. Prove that

$$f(X - f(X)) = f(X) - f(f(X)),$$

   where for a set S, the notation $f(S)$ means $\{f(a) \mid a \in S\}$.

4. Find all functions $f : \mathbb{N} \to \mathbb{N}$ such that for all $n \in \mathbb{N}$, the following holds:

$$f(1)^3 + f(2)^3 + ... + f(n)^3 = (f(1) + f(2) + ... + f(n))^2 .$$

5. Prove that for all polynomials $P \in \mathbb{R}[x]$ and positive integers n, $P(x) - x$ divides $P^n(x) - x$ as polynomials.

6. Find all polynomials f with real coefficients such that for all reals x, y, z such that $x+y+z = 0$, the following relation holds:

$$f(xy) + f(yz) + f(zx) = f(xy + yz + zx).$$

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Day 3
Time: 4.5 hours

1. Find all functions $f : \mathbb{Z} \to \mathbb{Z}$ such that

$$f(m+n) + f(mn-1) = f(m)f(n) + 2$$

for all $m, n \in \mathbb{Z}$.

2. *(Serbia MO 2016 P1)*

   (i) Does there exist a positive integer $m > 2016^{2016}$ such that $\frac{2016^m - m^{2016}}{m+2016}$ is a positive integer?

   (ii) Does there exist a positive integer $m > 2017^{2017}$ such that $\frac{2017^m - m^{2017}}{m+2017}$ is a positive integer?

3. Let $a, b, c \in \mathbb{R}^+$. Prove that

$$\sum_{cyc} ab \left(\frac{1}{2a+c} + \frac{1}{2b+c} \right) < \sum_{cyc} \frac{a^3+b^3}{c^2+ab}.$$

4. The cells of a 8×8 table are colored either black or white so that each row has a different number of black squares, and each column has a different number of black squares. What is the maximum number of pairs of adjacent cells of different colors?

5. Let $a, b, c \in \mathbb{R}^+$ such that $a + b + c = 3$. Prove that

$$\sum_{cyc} \left(\frac{a^3+1}{a^2+1} \right)^4 \geqslant \frac{1}{27} \left(\sqrt{ab} + \sqrt{bc} + \sqrt{ca} \right)^4.$$