

Thailand October Camp 2013
IPST, Bangkok
31 October 2013

Inequalities Exam
Time: 3 hours

1. Let x, y, z be positive real numbers. Prove that

$$4(x^2 + y^2 + z^2) \geq 3(xy + yz + zx).$$

2. Let a, b, c be positive real numbers. Prove that

$$\sqrt{\frac{a^2 + b^2}{a + b}} + \sqrt{\frac{b^2 + c^2}{b + c}} + \sqrt{\frac{c^2 + a^2}{c + a}} \geq \sqrt{\frac{2ab}{3a + b + 2c}} + \sqrt{\frac{2bc}{3b + c + 2a}} + \sqrt{\frac{2ca}{3c + a + 2b}}.$$

3. For positive real numbers a, b, c such that $abc = 1$, prove that

$$\frac{\frac{1}{a} + \frac{1}{b} + \frac{1}{c} + 1}{(a + b + c + 1)^2} + \frac{3}{8} \sqrt[3]{\frac{(a^2 - b^2)^3 + (b^2 - c^2)^3 + (c^2 - a^2)^3}{(a - b)^3 + (b - c)^3 + (c - a)^3}} \geq 1.$$

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Number Theory Exam
Time: 3 hours

1. Find all triples of positive integers (a, b, c) such that

$$(2^a - 1)(3^b - 1) = c!$$

2. **[sic]** Prove that there are infinitely many positive integer solutions to $x^8 = n! + 1$.
3. Let $s(n)$ denote the sum of digits of a positive integer n . Prove that $s(9^n) > 9$ for all $n \geq 3$.

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Combinatorics Exam
Time: 3 hours

1. Find the number of ways to put a number in every unit square of a 3×3 square such that any number is divisible by the number directly to the top and the number directly to the left of it, and the top-left number is 1 and the bottom-right number is 2013.
2. Find the number of permutations $(a_1, a_2, \dots, a_{2013})$ of $(1, 2, \dots, 2013)$ such that there are exactly two indices $i \in \{1, 2, \dots, 2012\}$ where $a_i < a_{i+1}$.
3. Let S be the set of all 3-tuples (a, b, c) of positive integers such that $a + b + c = 2013$. Find

$$\sum_{(a,b,c) \in S} abc.$$

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Algebra and Functional Equations Exam
Time: 3 hours

1. Find all functions $f : \mathbb{R}^+ \rightarrow \mathbb{R}^+$ such that

$$f(1 + xf(y)) = yf(x + y)$$

for all $x, y \in \mathbb{R}^+$.

2. In a triangle ABC , let $x = \cos \frac{A-B}{2}$, $y = \cos \frac{B-C}{2}$, $z = \cos \frac{C-A}{2}$. Prove that

$$x^4 + y^4 + z^4 \leq 1 + 2x^2y^2z^2.$$

3. Define $a_k = 2^{2^{k-2013}} + k$ for all integers k . Simplify

$$(a_0 + a_1)(a_1 - a_0)(a_2 - a_1) \cdots (a_{2013} - a_{2012}).$$

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Geometry Exam
Time: 3 hours

1. In a triangle ABC , $AC = BC$ and D is the midpoint of AB . Let E be an arbitrary point on line AB which is not B or D . Let O be the circumcenter of $\triangle ACE$ and F the intersection of the perpendicular from E to BC and the perpendicular to DO at D .
Prove that the acute angle between BC and BF does not depend on the choice of point E .
2. In a triangle ABC , the incircle with incenter I is tangent to BC at A_1 , CA at B_1 , and AB at C_1 . Denote the intersection of AA_1 and BB_1 by G , the intersection of AC and A_1C_1 by X , and the intersection of BC and B_1C_1 by Y . Prove that $IG \perp XY$.
3. Let O be the incenter of a tangential quadrilateral $ABCD$. Prove that the orthocenters of $\triangle AOB$, $\triangle BOC$, $\triangle COD$, $\triangle DOA$ lie on a line.