$\infty \cdot$ MO

Language: **English**

Day: **1**

Problem 1. We have a deck of $mn \geq 3$ cards. A *weird shuffle* is a process of shuffling a deck of cards following these steps:

1. Pull out the first m cards of the deck, making a pile. Repeat this until we get n piles.
2. Arrange the new deck by placing the top card of pile 1 to the *bottom* of the new deck, following by the top card of pile 2, that of pile 3 and so on. Continue doing this until placing the top card of the pile n, and then go back to that of pile 1. Repeat these processes until the cards in all n piles run out.

Show that the deck will return to the initial position in at most $mn - 1$ weird shuffles.

Problem 2. Determine all polynomials f with integer coefficients such that there exists an infinite sequence $a_1, a_2, a_3, \ldots$ of positive integers with the property: for all $n \in \mathbb{N}$, the sum of any $f(n)$ consecutive terms of this sequence is divisible by $n + 1$.
(Note: This requires $f(n) > 0$ for all $n \in \mathbb{N}$.)

Problem 3. Let $\triangle ABC$ be an acute triangle with circumcenter O and orthocenter H. The line through O parallel to BC intersect AB at D and AC at E. X is the midpoint of AH. Prove that the circumcircles of $\triangle BDX$ and $\triangle CEX$ intersect again at a point on line AO.

Time: 4 hours and 30 minutes
Each problem is worth 7 points

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Day: **2**

Problem 4. Given an acute triangle $\triangle ABC$ with circumcircle ω and circumcenter O. The symmedian through A intersects ω again at $S \neq A$. Point F is on AC such that $BF \perp AS$, and point G is on ray BF such that $BF \times BG = BC^2$. Finally, let P be the point such that $\square BGCP$ is a parallelogram. Prove that OS bisects CP.
(Note: The symmedian is the reflection of the median over the internal angle bisector.)

Problem 5. Suppose that we draw t straight lines through an $n \times n$ table such that for each unit square U in the table, at least one line passes through the *interior* of U.

Prove that $t > (2 - \sqrt{2})n$.

Problem 6. Given a polynomial $P \in \mathbb{R}[x]$ with odd degree. A real number x is called *orbiting* if the sequence

$$x,\ P(x),\ P(P(x)),\ \ldots$$

is bounded. Show that if every orbiting number is rational then there are finitely many (or zero) orbiting numbers.