J1. Let $\mathbb{Z}_{>1}$ denote the set of all integers greater than 1. Is there a function $f : \mathbb{Z}_{>1} \to \mathbb{Z}_{>1}$ such that

$$f^{f(n)}(m) = m^n$$

for all integers m, n greater than 1?

Note: for any positive integer k, $f^k(n)$ denotes the result of f being applied k times to n.

J2. Find all pairs (a, b) of positive integers such that $(a+1)^{b-1} + (a-1)^{b+1} = 2a^b$.

J3. There is a calculator with a display and two buttons: $-1/x$ and $x+1$. The display is capable of displaying precisely any arbitrary rational numbers. The buttons, when pressed, will change the value x displayed to the value of the term on the button. (The $-1/x$ button cannot be pressed when $x = 0$.)

At first, the calculator displays 0. You accidentally drop the calculator on the floor, resulting in the two buttons being pressed a total of N times in some order. Prove that you can press the buttons at most $3N$ times to get the display to show 0 again.

Note: partial credit will be given for showing a bound of cN for a constant $c > 3$.

J4. A sequence $a_1, a_2, \ldots$ of positive integers satisfies

$$a_n = \sqrt{(n+1)a_{n-1} + 1} \quad \text{for all } n \geqslant 2.$$

What are the possible values of a_1?

J5. A positive integer $n > 2$ is chosen, and each of the numbers $1, 2, \ldots, n$ is colored red or blue. Show that it is possible to color each subset of $\{1, 2, \ldots, n\}$ either red or blue so that each red number lies in more red subsets than blue subsets, and each blue number lies in more blue subsets than red subsets.

J6. Determine all positive reals r such that, for any triangle ABC, we can choose points D, E, F trisecting the perimeter of the triangle into three equal-length sections so that the area of $\triangle DEF$ is exactly r times that of $\triangle ABC$.

Time: 4 hours and 30 minutes
Each problem is worth 7 points